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So in the nude, in the free life and thought of the day, I would have you see what I mean by an immediate dualism. It is that walk in the forest with fear of a beast but always possible sight of a God. Somehow an unprotected life stands out as a protest, as a challenge, against any separation of the material and the spiritual; against a dual dualism and either of its acolytes, an abstract idealism or an abstract materialism. The free life is such a real and immediate struggle of body and soul! Almost too dramatic a theme for our ordinary philosophical jargon!

And, there being this struggle, so real and so immediate, it is all the more important in these times, not merely that professional philosophers, being realists, should be also immediate dualists, but also, as I have said already, that in this special era of philosophy and reconstruction with its reversion to the natural and the real, with its open intimacy of the ideal and the vital, the pending changes be consummated, while without oppression, with benefit of law and order. It can not properly be the part of philosophy to translate its realism into anarchy or its naturalism into a Garden of Eden. Rather must philosophy prove its appropriate heritage of self-control by realizing that violence and sudden change or revolution must delay if not defeat real progress. Progress, as demanded by the day's close struggle of ideality and vitality, can indeed be accomplished, for that matter even safety can be secured, only by enlightened and sympathetic thinking, by institutional generosity and by some real satisfaction of the new hopes and hungers. Dogmatic naturalism, throwing off all the protective covering of a hard won civilization, like its counterpart, obstinate conservatism, would bring only disaster. The great power of a great thinker is not more in his vision than in his self-control.

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THE CONCEPTS OF CLASS, SYSTEM, AND LOGICAL SYSTEM

THERE is still some disagreement as to the exact meaning of the concept "class." The older symbolic logicians, such as Boole and Schroeder, included under the term any collection whatsoever, regardless of whether or not the entities of which the collection was composed had any properties in common. "Cabbages and Kings" form a typical class in this sense of the word.

Modern logicians, however, under the leadership of the authors of *Principia Mathematica*, have somewhat modified this definition. Though they still maintain that a class is entirely determined once

its membership is fixed, they have added to the idea of class, as a mere collection, the requirement that the collection objects must have some common characteristic or fulfil some common condition. This has been done by defining the concept "class" by means of the idea of a propositional (or other) function, for which a class is the complete collection of (true) values. Without going into the subtleties of the definition as given in the *Principia*, particularly by neglecting the difficulties introduced by the fact that a class is defined only in use, the definition can be roughly put: that a class is an exclusive and complete collection of entities all of which fulfil some common condition. In this sense not every collection is a class, or, at least, there is no necessity imposed by the definition itself that every collection be a class. Moreover, though it is not necessary to specify completely the common condition needed to define a class, in practise this condition must be logically significant, thus excluding as classes collections the common condition of which are irrelevant to the universe of discourse in question. (To the *Principia* such a collection would be " $\phi, \epsilon, \mathbf{C}$ " and, perhaps, more important, excluding collections which are composed of entities of different types.)

Now, though it is necessary that the objects that form a class be an exhaustive list of the (true) values of some function, that is, have some characteristic in common, it must not be supposed that the nature of the function of which the entities are values affects the nature and properties of the class it defines, except in so far as it determines the membership. It happens that the pen, pencil and paper which form the class "objects on my desk," are the only writing implements in the room. They, therefore, constitute the class, "writing implements in the room," and this class, because it has the same membership as the class "objects on my table" is identical with it, *20.11¹, though it is hard to find anything that the defining functions have in common except that they happen to be satisfied by the same set of entities.

On the other hand, the fact that, even though according to the theory, the properties of any class are purely extensional, that is, are defined not as an arbitrary collection but as a collection satisfying some condition, gives the logician power to deal with a class even though he does not know what are, in detail, the values that define it, for he can always consider the class to be a collection of hypothetical values which have only the property that they satisfy the function in question. This makes possible the logical treatment of infinite and other classes, the membership of which can not be enumerated, and,

¹ Starred numbers refer to theorems or sections of the *Principia Mathematica*, Whitehead and Russell, Cambridge.

more important, allows the development of a theory of classes in general, which would obviously not be possible if all classes must be enumerated.

Now, no one would deny that the concept class as defined above is of very great importance to that part of logic and mathematics which deals with the most general relations of groups, regardless of their private nature. In fact, no more general type of collection could possibly be of much use to these sciences, for, even if such existed, it would at least be the exclusive and complete set of values of "the group which satisfies the logical condition in question," and hence be a class in spite of itself.

However, no matter how adequate the concept of greater concreteness than, say "quantity," there are certain special aggregates which no complete theory of the science can get along without, and which deserve definition. These are the concept of system and logical system. As we have said, one of the principal properties of classes is that two classes which have the same membership are identical, but, in ordinary life as well as in logic, we constantly meet useful aggregates which, though they have the same membership, have quite different properties, considered as a collection. Thus the collection of raw recruits out of which an army is made is quite different, as a collection, from the army after organization, though of course, recruits and army form quite the same class. Thus a crystal of common salt is quite a different aggregate from the mixture of sodium and chlorine into which it can be dissociated, though both crystal and mixture, being composed of the same atoms of Na and Cl, are the same class. For, as we can easily generalize, the concept of class is not adequate to deal with aggregates the significant properties of which depend on the organization of their parts, that is, depend on the relation between their members. Yet internally related aggregates exist in which the resultant properties are a function of the nature of the relation, and these aggregates are of great practical as well as logical interest, being what are commonly called systems. We may, therefore, define a system as an aggregate such that each of its members has a definite relation to some other member or group of members of the aggregate. Or, in the nomenclature of the *Principia*, a system may be defined as the class which is the field of a certain sort of relation, *i. e.*,

$$\text{Sys} = \hat{\alpha}\{(\exists \mathfrak{R})_{\alpha \in C} \mathfrak{R}\} \quad Df$$

where the symbols have their usual meaning except we have a different R than that used in the *Principia* to indicate that "relations" can not have the extensional meaning (*i. e.*, a class of couples) given to them in the *Principia*; otherwise, by *33.45-6, two systems with the same defining relations would be identical, and one of the chief

formal differences that distinguish a system from a mere class must be that if a class of members of two systems are identical while the defining relations are different, the systems must be different.

Moreover, the fact that an extensional logic such as that of the *Principia* is, as far as I can see, unable properly to define a system is no mere omission on its part. Every possible logic, extensional or otherwise, every bit of connected or even intelligible discourse, is a system, in the sense defined above, since it differs obviously from the mere collection of its theorems, in that proofs are present, and thus any logic which considers terms to be defined by extension is logically imperfect, since its own terms are not so defined. The system composed of theorem and postulate, so related that every theorem is proven, can, according to the extensional theory, differ in no way from the disordered group of theorems and postulates which are merely asserted, and thus, extensional logic itself differs in no way from the mere dogmatic statement of its unproven theorems.

This same point comes out even more strongly in connection with the third of the concepts which we wish to define: that of logical system. A logical system does not differ from a system as defined above in any such fundamental way as a system differs from a class. Logical systems are merely a particular class of systems; nevertheless they are of importance, as can be most readily seen by attempting to differentiate such a system as logic from a less logical treatise composed of the same terms. Without some specification of the general nature of the relation which makes it a system, it is impossible to give the general theory of the distinction between a related list, where the relation is merely an arbitrary one, and such a related structure as logic, where, given one term or group of terms, all of the other terms are determined. Such systems, of which the external world and science are the most important examples, seem to deserve separate treatment under the name of logical systems. Loosely we may define a logical system as a system such that the properties of a separate part determines or implies the properties of the remainder. Or, more formally, a logical system is a system such that some of its members are unambiguously related to other of its members, where an unambiguous relation is any relation such that when the relata are given the relatum is completely determined. In effect then, a logical system is a system which can be considered under the postulate-theorem form, though logical systems are by no means limited to any special kind of membership (such as propositions) since, as we have said, both the causal system of the external world and the true account of it are logical systems, though they have memberships which do not even belong to the same type, one set of membership things and the other propositions.

We do not wish, however, to elaborate the connection of logical systems with the problem of types. We wish merely to point out that logical systems exist, and that all logic and all science are necessarily examples of them, and also to point out that no purely extensional logic can account for the existence of logical systems or their properties, thus placing extensional logic in the uncomfortable position of not being able to account for the very characteristic, namely, that theorem unambiguously follows from postulate, which makes it a science at all.

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NOTE ON THE RELATION OF SUBALTERNATION

IN a recent article in this JOURNAL (*Non-Aristotelian Logic*, August 15, 1918), in which a generalization of the classical logic was proposed, the relations of subalternation were tacitly held to be true.

This feature of the science being all but universally denied in recent times,¹ it was not unnatural that a number of critics should have privately informed the writer that this assumption invalidated some of his results.²

Thus, if we employ the symbol, \angle , for *inclusion*, the four categorical forms, A, E, I, O, might supposedly be represented as follows (b' standing for non- b ; the "prime" to the right of the bracket indicating that the proposition is *false*):

- (A) All a is $b = (a \angle b)$
- (E) No a is $b = (a \angle b')$
- (I) Some a is $b = (a \angle b)'$
- (O) Some a is not $b = (a \angle b)'$

¹ Cf. Couturat (*Des propositions particulières*, *Revue de Métaphysique et de Morale*, t. XXI., p. 258).

"Du moment que les particulières sont des existentielles négatives, on ne peut pas déduire une particulière d'une universelle (ni inversement). Donc la subalternation classique est fautive. De: «Il n'y a pas de a non- b » on ne peut nullement inférer: «Il y a des ab ». Cette inférence n'a pu faire illusion que grâce à la prémisse additionnelle et tacite: «Il y a des a », qui semblait impliquée dans le langage."

Couturat in the same article (p. 257) attaches the following meaning to A, E, I and O:

- (E) Nul a n'est $b =$ Il n'y a pas de ab .
- (A) Tout a est $b =$ Il n'y a pas de a non- b .
- (I) Quelque a est $b =$ Il y a des ab .
- (O) Quelque a n'est pas $b =$ Il y a des a non- b .

² It was this misapprehension, which the original article ought to have removed; but what follows will serve to present the matter from another point of view.